

BINARY PHASE

SHIFT KEYING

(BPSK)

DIGITAL
MODULATION.

BPSK. Pulse Shape $p(t)$

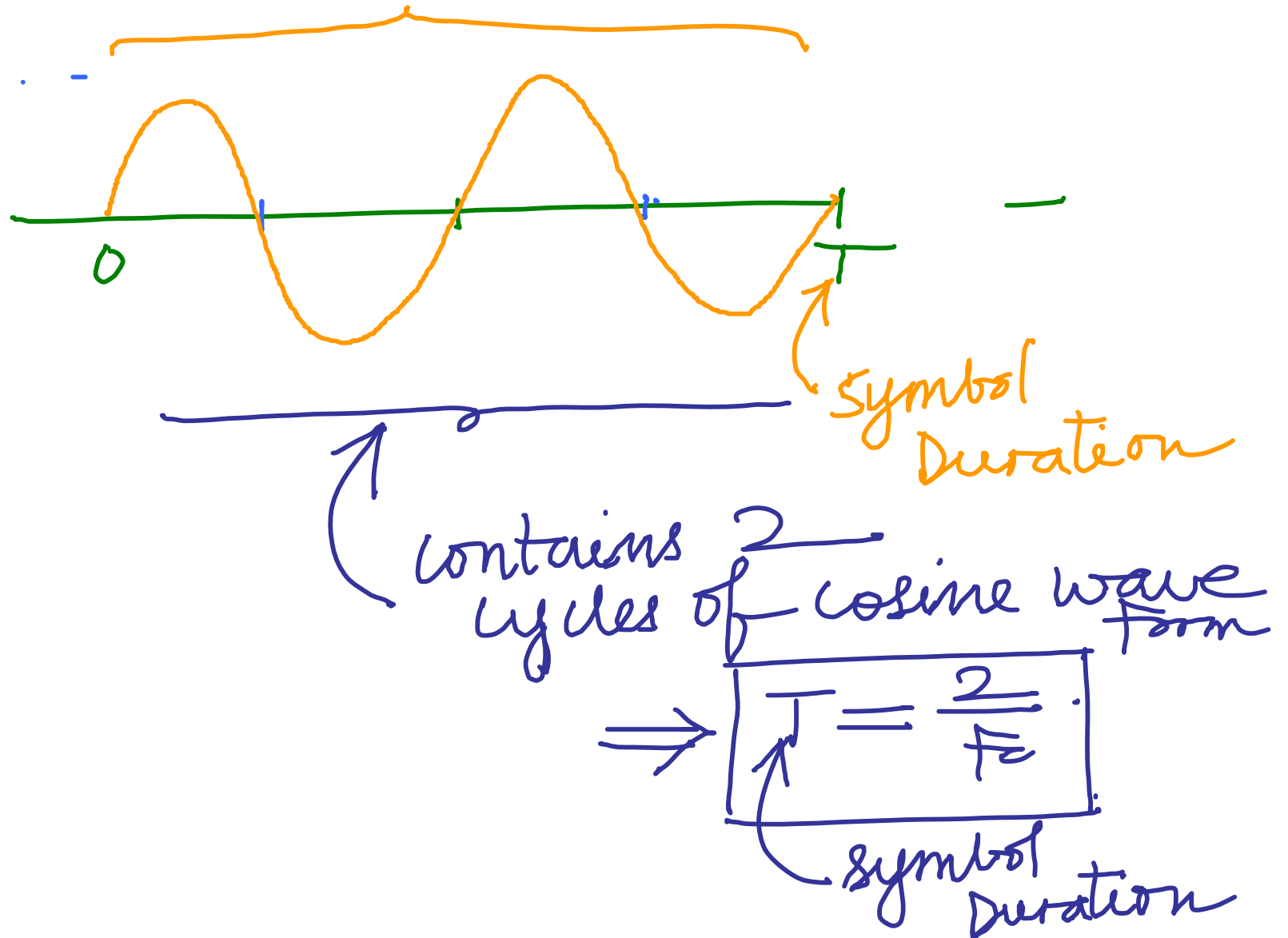
$$p(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$0 \leq t \leq T$$

Symbol
Duration

$T =$ integer multiple
of $\frac{1}{f_c}$
contains an integer number of cycles of $\cos(2\pi f_c t)$
 $T = \frac{k}{f_c}$ ← integer

$$p(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$



$p(t)$ ← Normalized to
Unit Energy.

$$E_p = \int_{-\infty}^{\infty} p^2(t) dt$$

$$= \int_0^T \frac{2}{T} \cos^2(2\pi f_c t) dt$$

$$= \frac{2}{T} \int_0^T \frac{1 + \cos 4\pi f_c t}{2} dt$$

$$= \frac{2}{T} \cdot \frac{T}{2} + \frac{1}{T} \int_0^T \cos(4\pi f_c t) dt$$

$$= \boxed{1. = \epsilon_p.}$$

$$X(t) = a_0 p(t)$$

$$a_0 \in \{-A, A\}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ 1 & 0 \end{array}$$

Each a_0 carries
1 bit of information

$\epsilon_b =$ energy/bit
constant across
schemes.

information bits $\left\{ \begin{array}{l} 0 \text{ — occurs with prob } \frac{1}{2} \\ 1 \text{ — occurs with prob } = \frac{1}{2} \end{array} \right.$

$$a_0 = \pm A$$

$$a_0 p(t) = x(t) \quad \text{Prob bit} = 1$$

average energy/bit

$$= \frac{1}{2} A^2 \epsilon_p + \frac{1}{2} A^2 \epsilon_p$$

Average Energy of Signal if 0 occurs.

Prob bit = 0 $\rightarrow$ $= A^2 \epsilon_p = A^2$ $\epsilon_p = 1$

$$A^2 = E_b.$$

$$\Rightarrow A = \sqrt{E_b}.$$

Ensures average energy/bit = E_b .

Therefore, the transmitted waveforms are,

$$0 \rightarrow a_0 = +A = \sqrt{E_b}.$$

$$x(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad 0 \leq t \leq T$$

$$= \boxed{\sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t)}$$

Waveform
corresponding to bit
Zero.

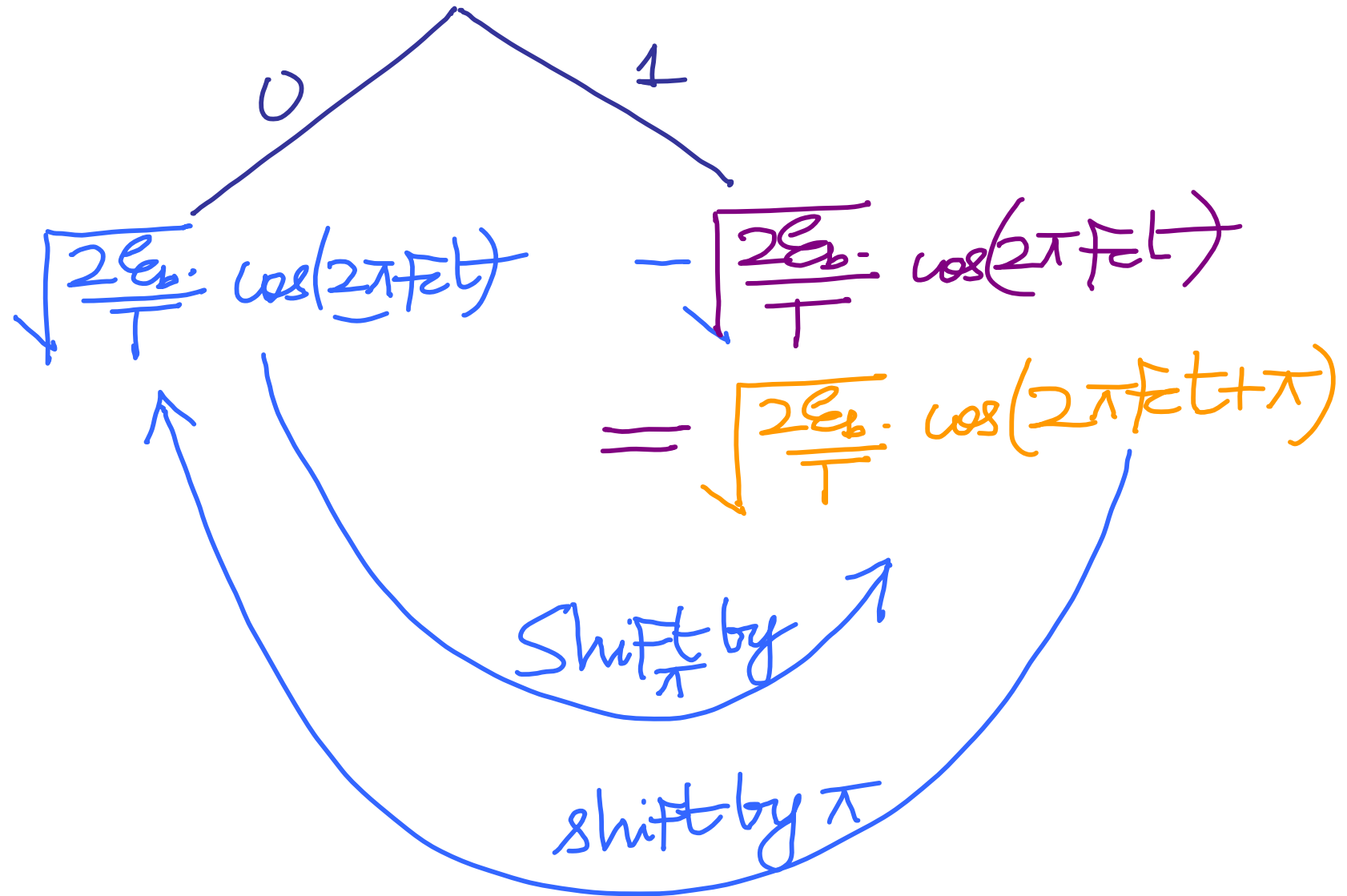
$$1 \rightarrow a_0 = -A = -\sqrt{E_b}.$$

$$\Rightarrow x(t) = -\sqrt{E_b} \cdot \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$= \boxed{-\sqrt{\frac{2E_b}{T}} \cos 2\pi f_c t}$$

Waveform
for bit 1.

Wave Forms



$$\sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + \pi + \pi)$$
$$= \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t)$$

2 wave forms $\Rightarrow$ Binary Modulation Scheme

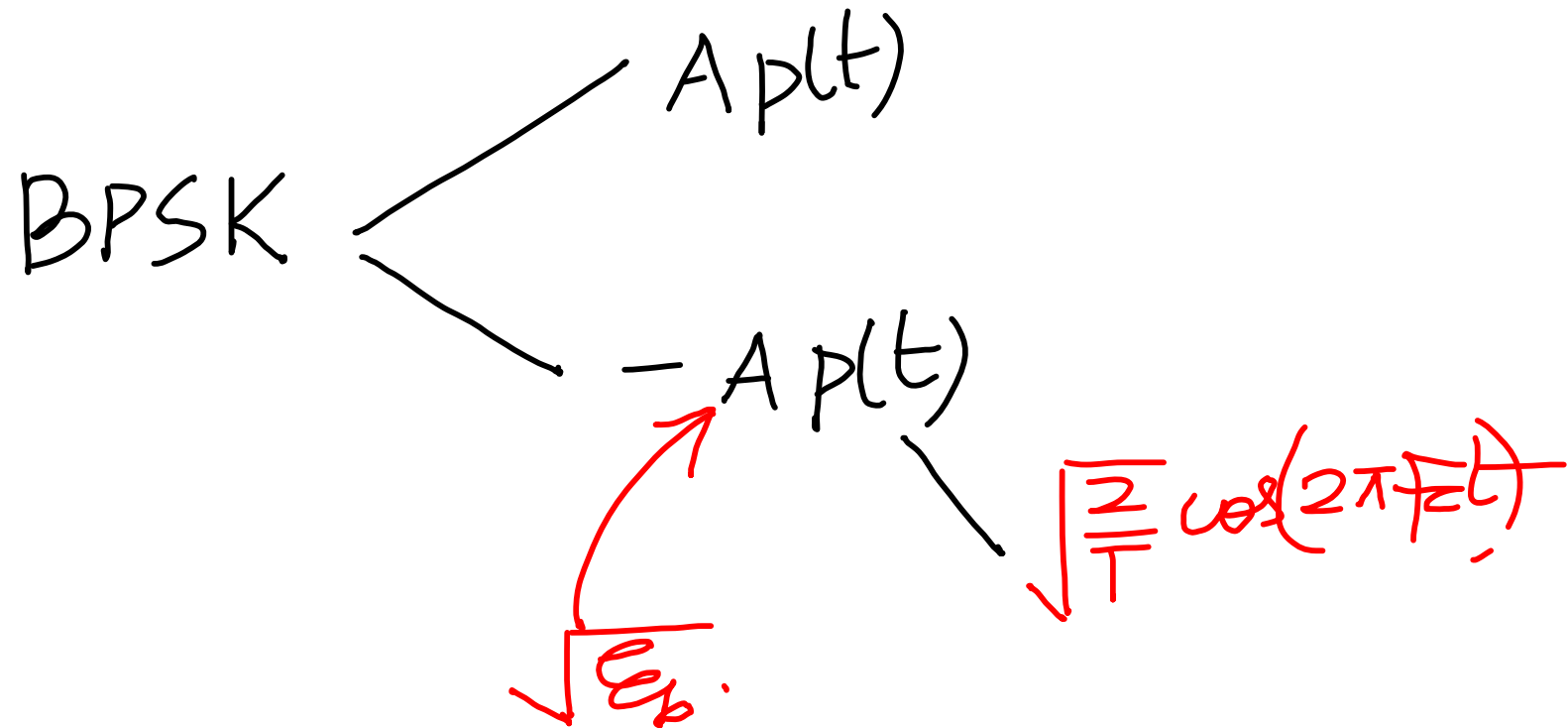
+

wave forms are
phase shifted wrto
each other $\Rightarrow$

+

Phase
Shifting

Binary Phase
Shift Keying
(BPSK)

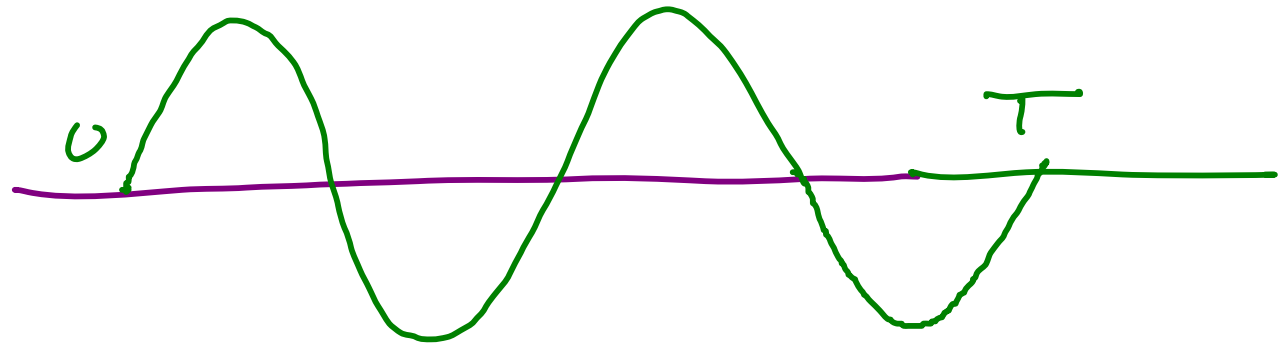
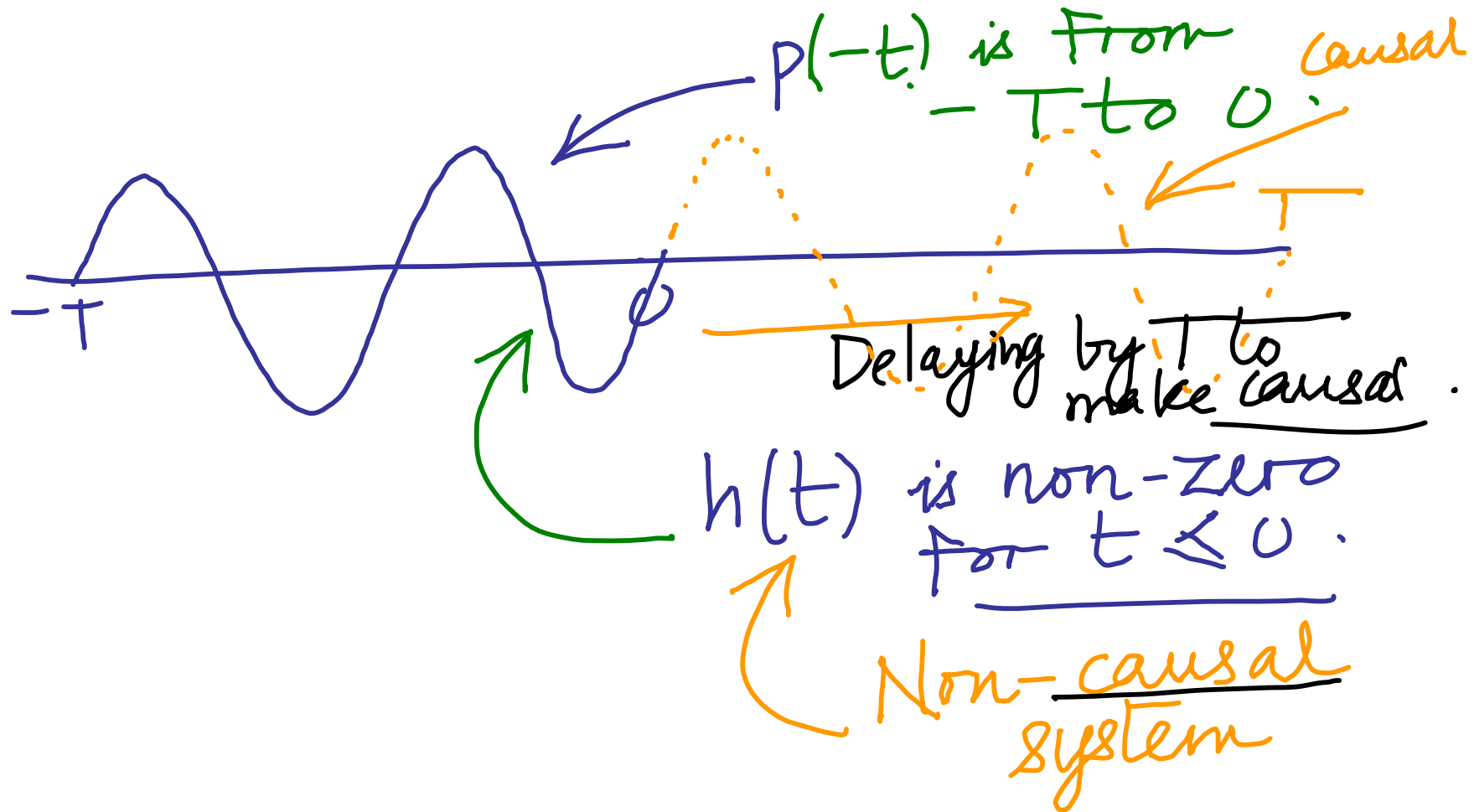


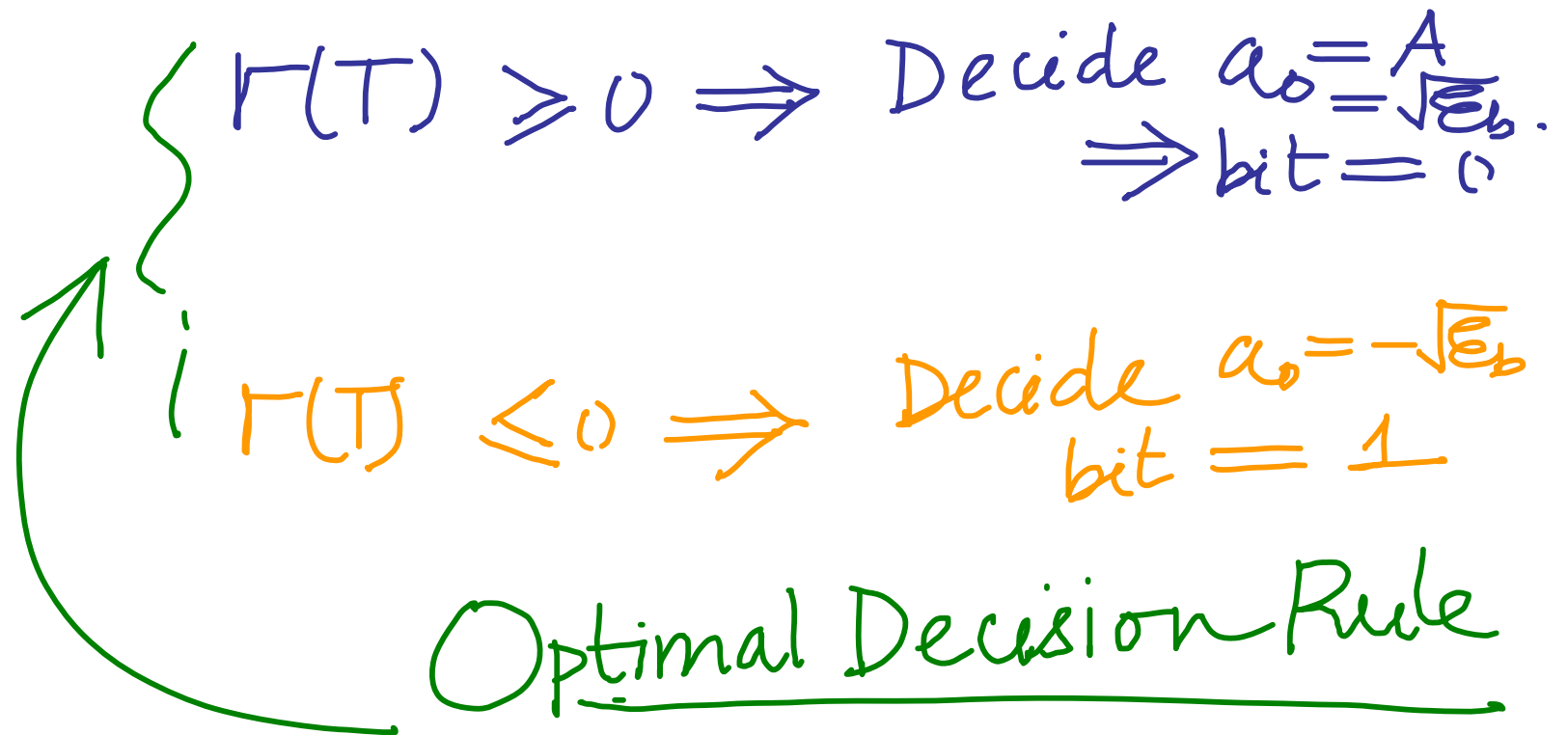
At receiver, matched Filter

$h(t) = \underline{p(T-\tau)}$

Filter by $h(t)$
Sample by T

$p(-\tau)?$





Prob of Error = $Q\left(\sqrt{\frac{A^2 E_p}{N_0/2}}\right)$
 Set $A = \sqrt{E_b}$
 $E_p = 1$

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0/2}}\right)$$

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Probability of Error
for BPSK, average
energy per bit = E_b .

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

↙ Denotes Probability
That standard Gaussian
with mean = 0, var = 1
is greater $\geq x$.